Chapter 12 - Spur Gears

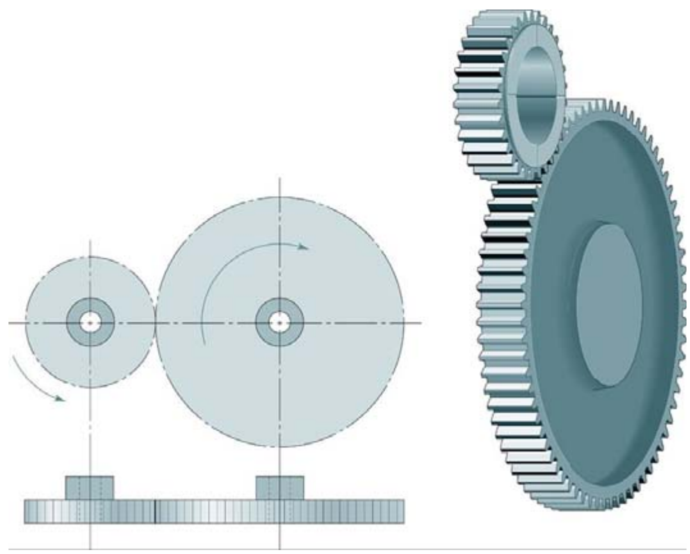
Gears have been used since ancient times to transmit rotary motions and torque from one shaft to another. In about 2600 B.C. the Chinese designed a chariot incorporating gears whose teeth were round wooden pegs stuck into the rims of the cylinders. Aristotle, in the 4th century B.C. mentioned gears in his writings and in the 15th century A.D., Leonardo da Vinci designed several devices incorporating many kinds of gears.

Gears are the most rugged and durable means of mechanical power transmission. Their power transmission efficiency could be as high as 98%. Gears, however, are usually more costly than other means of power transmission such as belts and chains. Gear manufacturing costs rise sharply with increased precision (for high speed and heavy loads), and for low noise levels. The standards for gears of any kind have been established by the AGMA, American Gear Manufacturers Association.

Types of Gears

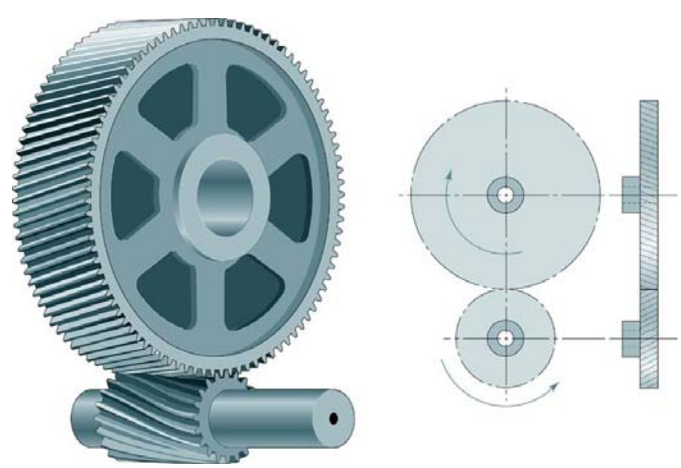
There are four principal types of gears: spur, helical, bevel, and worm gears.

Spur gears have teeth parallel to the axis of rotation and are used to transmit rotary motion between parallel shafts. The spur gear might be called the "basic gear" as all the other gears were developed from it.

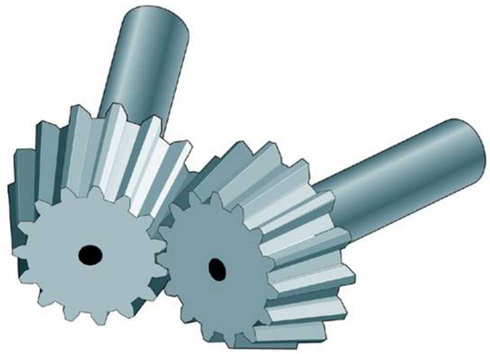


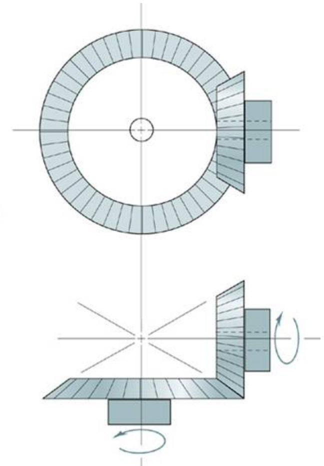
Helical gears have teeth inclined to the axis of rotation and are used to transmit motion between parallel as well as non-parallel shafts.

Because of the more gradual engagement of the teeth they are not as noisy as spur gears. The inclined teeth also develops thrust load which is not present in spur gears.



Bevel gears have teeth formed on conical surfaces and are used mostly for transmitting motion between intersecting shafts. There are "straight-tooth," "spiral," and other types of bevel gears.





Worms and **worm Gears** are as shown, The worm resembles a screw and just like screws, it may be left, or right-handed. So the direction of rotation of the worm gear, also called the worm wheel, depends upon the direction of the rotation of the worm and whether or not the worm teeth are right- or left-handed.

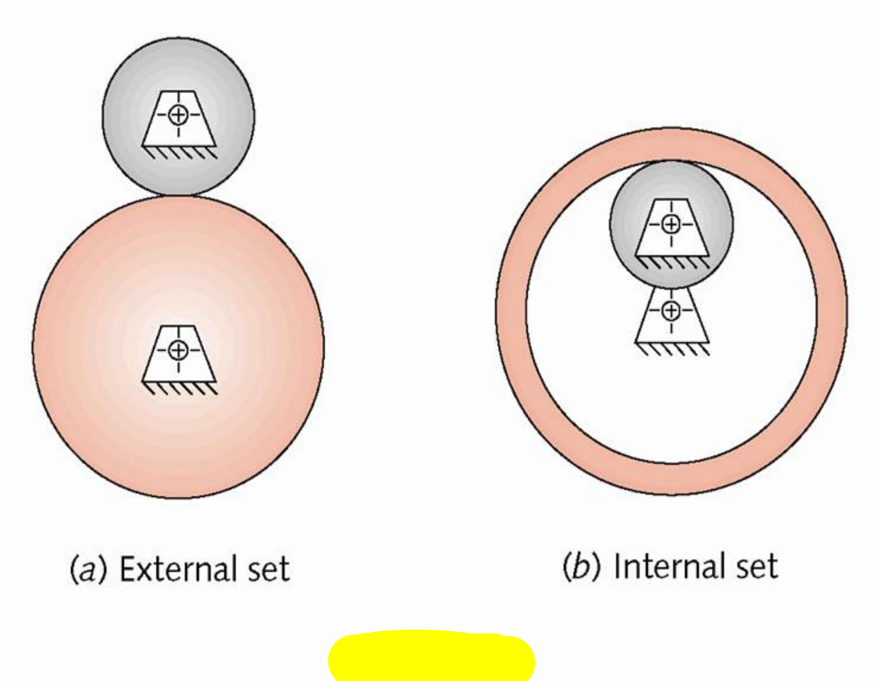
Worm gears are mostly used when the speed ratios are quite high, say, three or more.

(a)

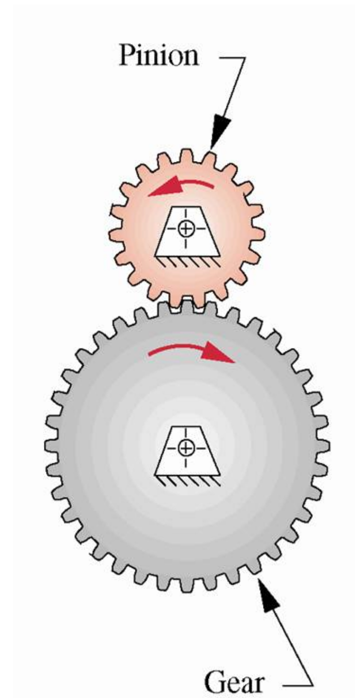
(b)

12.1 (Spur) Gear Tooth Theory

The simplest way of transferring rotary motion between two shafts is a pair of rolling cylinders. There will be no slip between the cylinders until the frictional force at the point of contact is overcome by the tangential force produced by the applied torque. Some drives, however, require the input and output shafts to be in-phase for timing purposes. To this end, some meshing teeth are added to the rolling cylinders which will make them a gearset. The smaller of the two gears is referred to as the pinion and the other as the gear, On large, heavy-duty drives, the large gear is called a bullgear.



Rolling Cylinders.



An External Gearset.

The Fundamental Law of Gearing

The shape of teeth should be such that it allows a smooth angular velocity transmission between the gears. Hence, there is a fundamental law of gearing that can be stated as follows:

> **The angular velocity ratio between the gears of a gearset must remain constant throughout the mesh.**

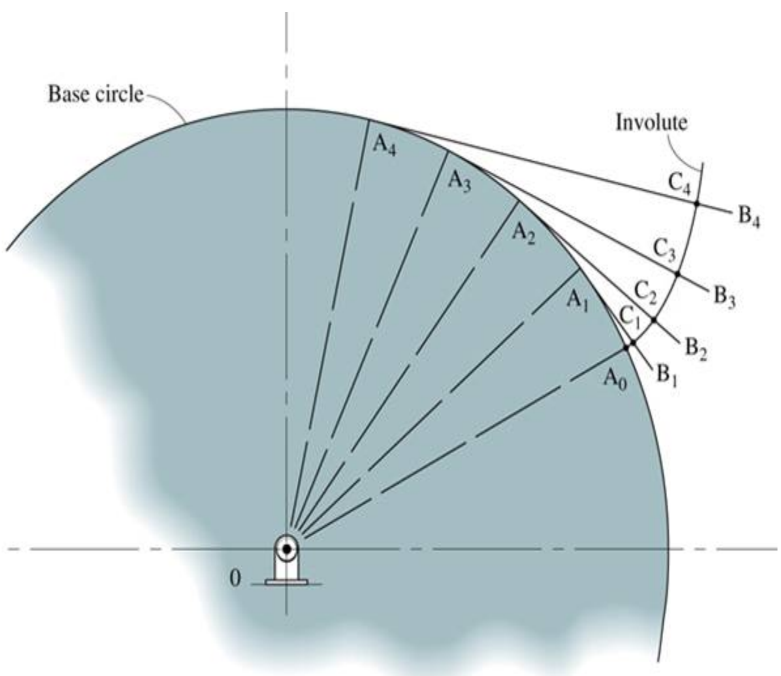
In order for this law to be true, the gear tooth contours on mating teeth must be conjugates of one another. There is an infinite number of conjugate forms that could be used, but only a few curves have been practical to use as tooth form. The cycloid is used as a tooth form in some clocks and watches, but most gears have their teeth in the shape of the involutes of a circle.

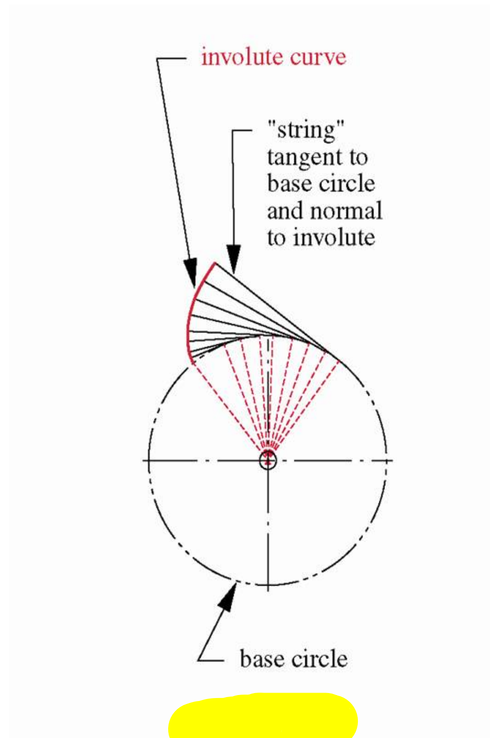
The Involute Tooth Shape

The involutes of a circle is a curve that can be generated by unwrapping a taut string from a cylinder. Some properties:

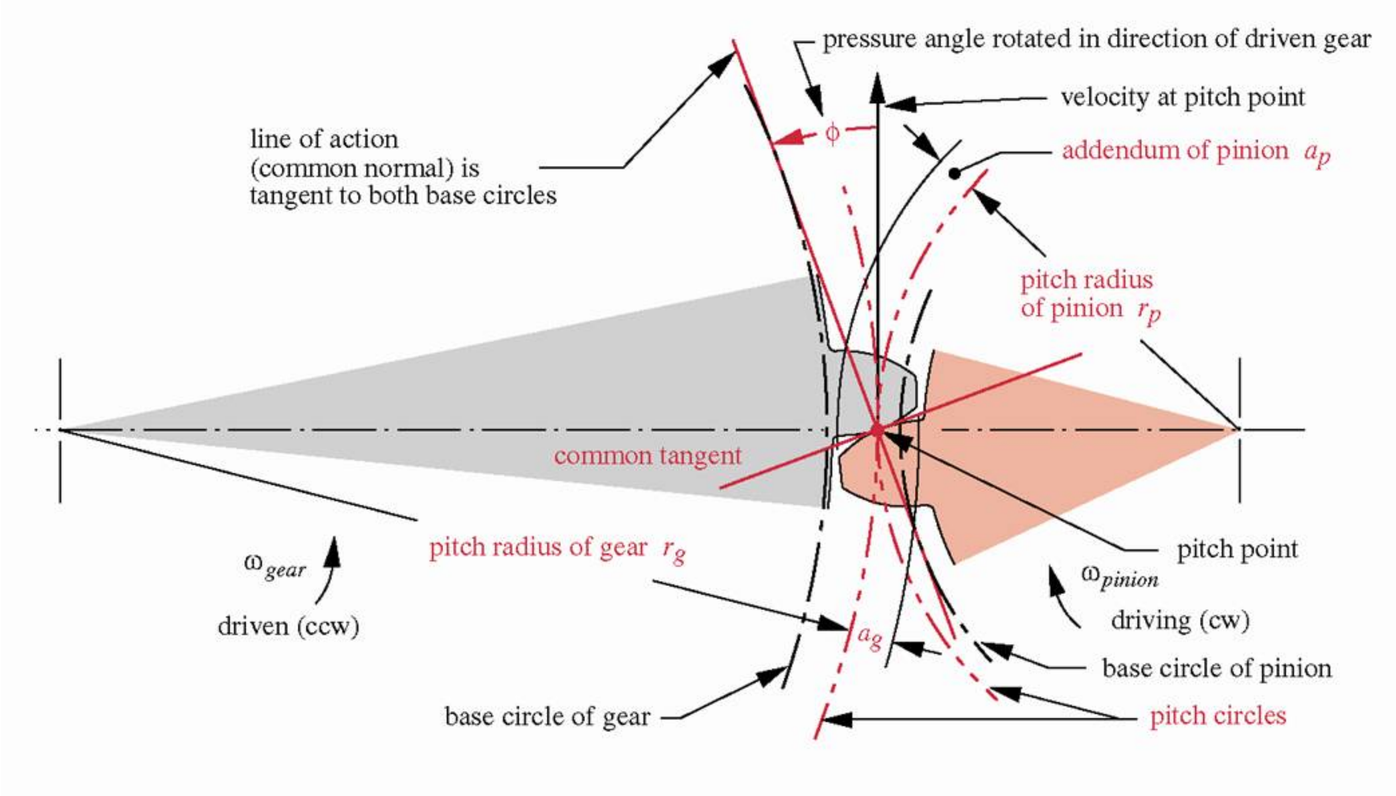
1. The string is always tangent to the circle.
2. The center of curvature of the involute is always at the point of tangency of the string with the base circle.
3. A tangent to the involute is always normal to the string.

Fig. 12-4 shows two involutes (gear teeth) "in contact" or "in mesh."





Development of the Involute of a Circle.



Contact Geometry and Pressure Angle of Involute Gear Teeth.

Base Circles: The cylinders from which the strings are unwrapped.

Pitch Circles: The surfaces of the rolling cylinders.

Pitch Point: The contact point between pitch circles which lies on the "line of centers."

Pitch Diameters: Diameter of pitch circles.

Pitch Radii (r_p and r_g): The radii of the original rolling cylinders.

Addendum (a_p and a_g): The amount of tooth that sticks out above the pitch circle.

Line of Action (Common Normal): This is perpendicular to the common tangent and passes through the pitch point.

Pitch-line Velocity (V_t): The linear velocity of the pitch point in both pinion and gear.

Pressure Angle, φ: The angle between the line of action and the velocity vector. Pressure angles are standardized at the values 14.5°, 20°, and 25° with 20° being the most commonly used and 14.5° now being obsolete. Any custom pressure angle can be made but it would be expensive. Gears to be run together must have the same pressure angle.

Angular Velocity Ratio: $m_V = \omega_{out} / \omega_{in} = \pm\, r_{in} / r_{out}$ (12.1a)

where r_{in} and r_{out} are the pitch radii

"-" is for external gearsets

"+" is for internal gearsets

Fundamental law of gearing: m_V = constant throughout the mesh.

Torque Ratio = Mechanical Advantage: $m_A = 1 / m_V$ (12.16)

Gear Ratio: $m_G = \begin{cases} |m_V| \\ \text{or} \\ |m_A| \end{cases}$ for $m_G > 1$ always (12.10)

In Fig. 12-5, one tooth is just beginning contact and the other is about to leave contact. The common normal, of both these contact points still pass through the same pitch point. It is this property that causes the involutes to follow the fundamental law of gearing (the ratio of the driving-gear radius to the driven gear radius remains constant as the teeth move into and out of mesh).

$$\text{Length of Action, } Z = \sqrt{(r_p + a_p)^2 - (r_p \cos\phi)^2} + \sqrt{(r_g + a_g)^2 - (r_g \cos\phi)^2} - C \sin\phi$$

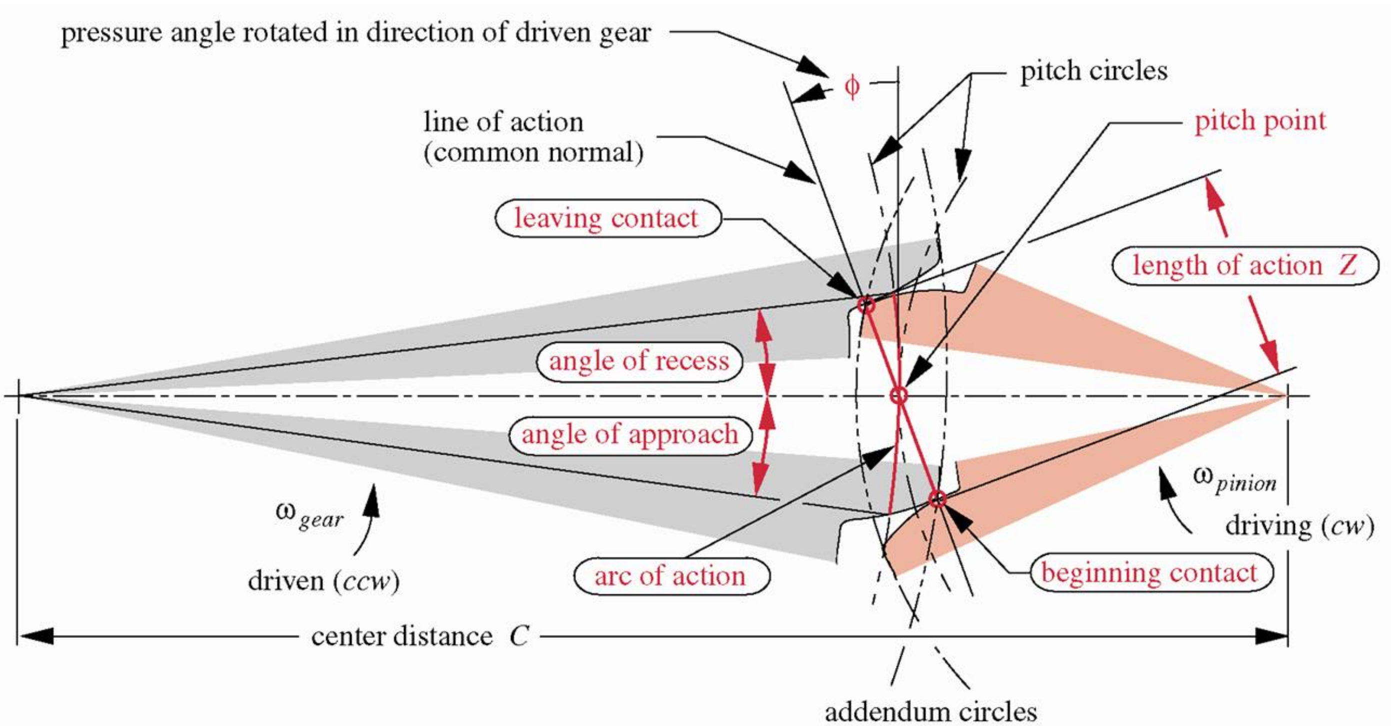
where r_p and r_g are the pitch radii of pinion and gear

a_p and a_g are the addenda of pinion and gear

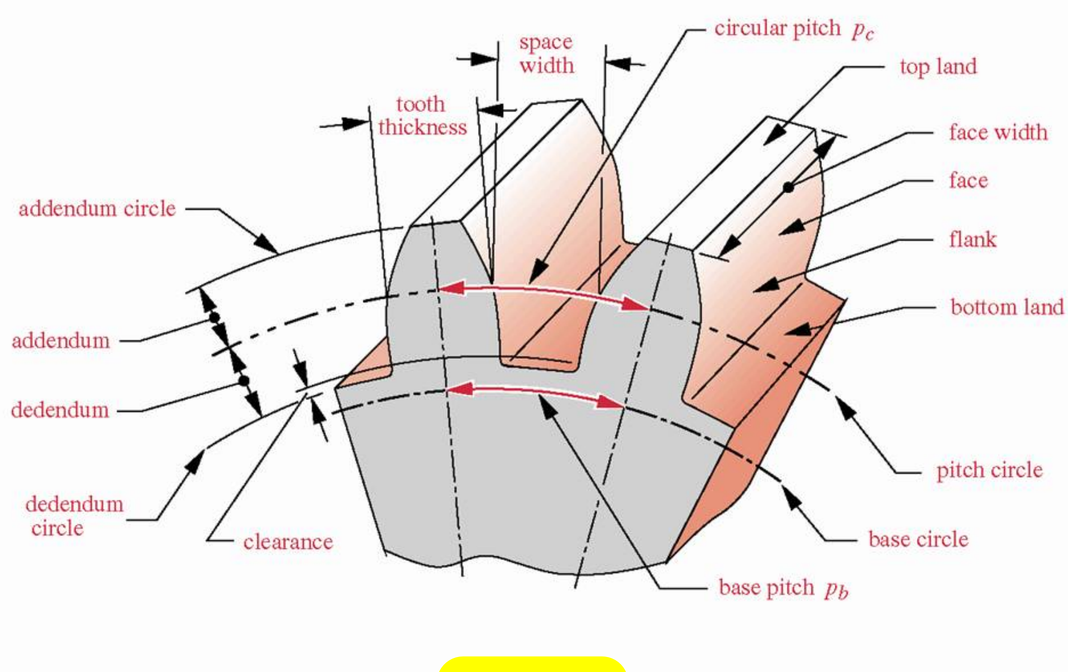
C is the center distance and φ is the pressure angle

Arc of Action: The distance along the pitch circle within the mesh.

Angle of Approach and **Angle of Recess**: The angles subtended by the beginning and end of mesh and the center line.



Length of Action, Arc of Action, and Angles of Approach and Recess During the Meshing of a Gear and Pinion.



Gear Tooth Nomenclature.

12.2 More Gear Tooth Nomenclature

Addendum (added on) and dedendum (subtracted from) represent the tooth height. The dedendum is slightly larger than the addendum to provide clearance between the tip of one tooth and the root of the tooth of the other.

Working depth is twice the addendum.

Whole depth is the sum of the addendum and dedendum.

Tooth thickness is measured at the pitch circle.

Space width is slightly larger than the tooth thickness.

Backlash is the difference between space width andtooth thickness.

Face width is the width measured along the axis.

Circular pitch, p_c, is the arc length along the pitch circle measured from a point on one tooth to the same point on the next. We have

$$p_c = \pi d / N \qquad (12.3a)$$

where d is the pitch diameter, and
N is the number of teeth

Base pitch, p_b, is similar to the circular pitch except that base circle is used instead of pitch circle. We have

$$p_b = p_c \cos \varphi \qquad (12.3b)$$

Diametral pitch, $p_d = N / d = \pi / p_c$

where (12.3a) has been used in the second equation.
p_d is used only in the U.S.

Module: $m = d / N$ measured in mm is used for metric gears in the SI system.

***Remark*: Metric & U.S. gears are not interchangeable. They both use involute forms but tooth sizes are different. The conversion from one standard to the other is**

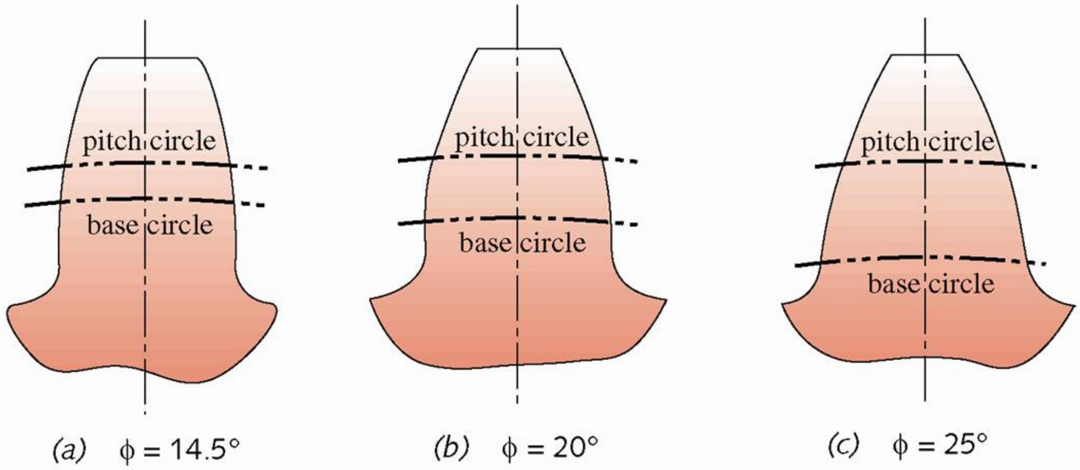
$$m = 25.4 / p_d \qquad (12.4d)$$

Since the diametral pitch, p_d of meshing gears are the same, the velocity ratio, m_V, defined by (11.1) is

$$m_V = \pm r_{in} / r_{out} = \pm (d_{in} / d_{out}).(p_d / p_d) = \pm N_{in} / N_{out} \qquad (12.5a)$$

Gear ratio: $m_G = N_g / N_p \qquad (12.5b)$

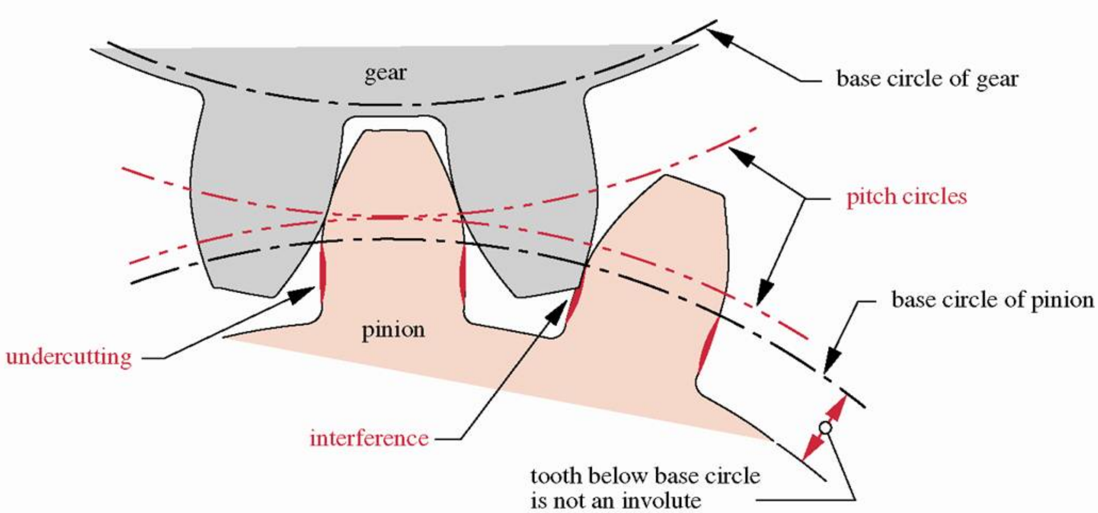
Standard Gear Teath - standard, full-depth gear teeth have equal addenda on pinion and gear, with the dedendum being slightly larger for clearance (see text for specifications).



AGMA Full-Depth Tooth Profiles for Three Pressure Angles.

12.3 Interference and Undercutting

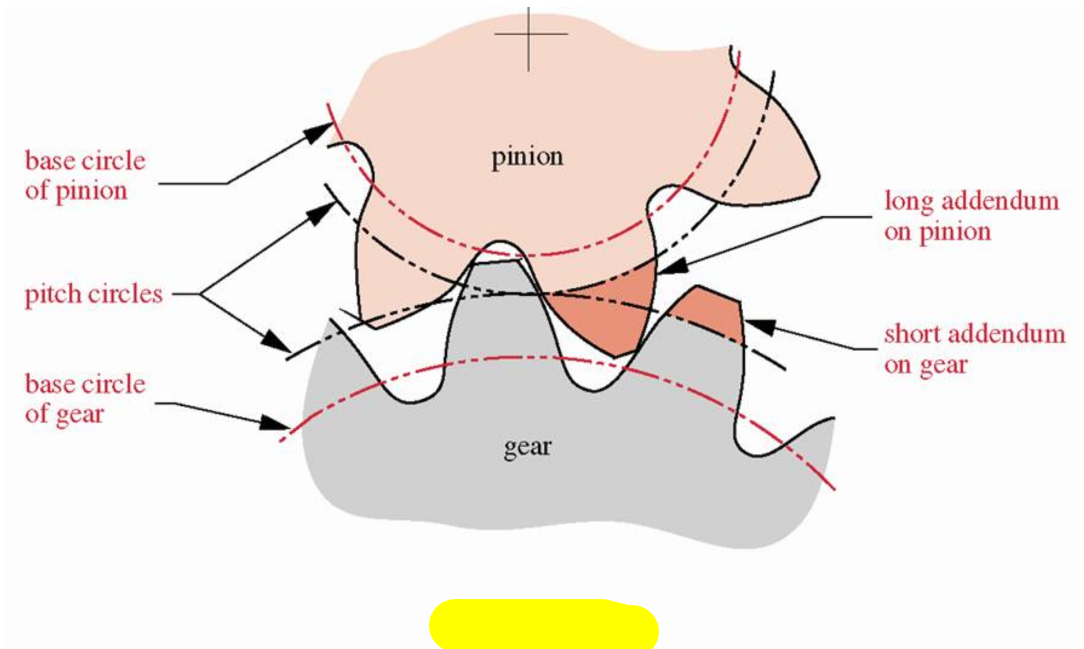
The contact of portions of tooth profile that are not an involute is called interference (the involute tooth form is only defined outside of the base circle). The effect of interference is called undercutting. If undercutting is pronounced, the undercut tooth will be weakened. Hence, interference & undercutting should be avoided.



Interference and Undercutting of Teeth Below the Base Circle.

Unequal-Addendum Tooth Forms

To avoid interference on small pinions, the tooth form can be changed from full-depth shape of Fig 12-9 that have equal addenda on both pinion and gear to an involute shape with a longer addendum on the pinion and a shorter one on the gear. These are called profile-shifted gears. The standard addition/ subtraction are 25% and 50% of the standard addendum, respectively.



Profile-Shifted Gear Teeth with Long and Short Addenda to Avoid Interference and Undercutting.

12.4 Contact Ratio

The average number of teeth in contact at any one time is called the contact ratio and is designated by m_p:

$$m_p = Z / p_b \qquad (12.7a)$$

where Z is the length of action (Eq. 12.2) and p_b is the base pitch (Eq. 12.3b). Employing (12.3b) and (12.4b), we find

$$m_p = Z\, p_d / (\pi \cos \varphi) \qquad (12.7b)$$

If $m_p = 1$, then one tooth is leaving contact just as the next is beginning contact. This is undesirable because the load will be applied at the tip of the tooth creating the largest possible bending moment. When $m_p > 1$, there is the possibility of load sharing among the teeth. Even for $1 < m_p < 2$ which are common for spur gears, depending on manufacturing precision, there will be times when the entire load is carried by one pair of teeth. However,

the load will be applied at a lower position on the tooth, rather than at its tip. This point of application of the applied load is called the Highest Point of Single-Tooth Contact or HPSTC.

The minimum acceptable contact ratio is 1.2. A minimum contact ratio of 1.4 is preferred, and larger is better.